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ACCURACY ASSESSMENT OF DIFFERENT POLYNOMIAL GEOID MODELS IN ORTHOMETRIC HEIGHT DETERMINATION FOR AKURE, NIGERIA

PROCJENA TAČNOSTI RAZLIČITIH POLINOMSKIH GEOIDNIH MODELA U ORTOMETRIJSKOM ODREĐIVANJU VISINE ZA PODRUČJE AKURE, NIGERIJA

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ABSTRACT

Height is an important component in three dimensional coordinates and determination of the position of points for any meaningful development. Ellipsoidal heights from GNSS require geoid model which could be global, regional and local for transformation to orthometric height. The absence of a national geoid model in Nigeria remains a great drawback to develop local geoid for local application in place of global geoid models. The study aims to assess the accuracy of polynomial geoid models in orthometric height determination. Differential Global Positioning System (DGPS) observations were carried out to determine ellipsoidal heights of the point while nine and eleven coefficients were used for the geoid and orthometric height modelling. Model A and Model C used 2-D (x, y) positions with nine and eleven parameters while model B used 3-D (x, y, Δh) positions with nine parameters. The least-squares method was adopted in computing the parameters of the models. Root Mean Square Error (RMSE) was used to assess the accuracy of the models with the RMSE of model A is 14.3 cm, model B is 15.7 cm and model C is 14.5 cm, respectively. The inclusion of height term (Δh) in model B does not improve the accuracy over model A and model C. Model A with the lowest RMSE is hence the better of the three models. One-way ANOVA test conducted at 95% confidence level,

SAŽETAK

Visina je važna komponenta u trodimenzionalnim koordinatama i određivanju položaja tačaka za svaki smisleni razvoj. Elipsoidne visine iz GNSS-a zahtijevaju geoidni model koji može biti globalni, regionalni i lokalni za transformaciju u ortometrijsku visinu. Odsustvo nacionalnog modela geoida u Nigeriji ostaje veliki nedostatak za razvoj lokalnog geoida za lokalnu primjenu umjesto globalnih modela geoida. Studija ima za cilj procijeniti tačnost polinomskih modela geoida u ortometrijskom određivanju visine. Opažanja Diferencijalnog Globalnog Pozicionog Sistema (DGPS) su obavljena s ciljem odrređivanja elipsoidnih visina tačaka, dok je devet i jedanaest koeficijenata korišteno za geoidno i ortometrijsko modeliranje visine. Model A i model C koristili su 2-D (x, y) pozicije sa devet i jedanaest parametara, dok je model B koristio 3-D (x, y, Δh) položaje sa devet parametara. U proračunu parametara modela usvojena je metoda najmanjih kvadrata. Korjen srednje kvadratne greške (RMSE) je korišten za procenu tačnosti modela. RMSE modela A je 14,3 cm, modela B je 15,7 cm i modela C je 14,5 cm, respektivno. Uključivanje pojma visine (Δh) u model B ne poboljšava tačnost u odnosu na model A i model C. Model A sa najnižim RMSE je stoga bolji od tri modela. Međutim, jednosmjerni ANOVA test sproveden na nivou pouzdanosti od 95% otkrio je da se tri modela *however, revealed that the three models did not differ significantly. Model A having lower RMSE is recommended with GPS determined ellipsoidal heights as an alternative to conventional spirit levelling for orthometric height determination within Akure for engineering and environmental applications.*

Keywords: DGPS, Ellipsoidal heights, Least-square, Orthometric heights, Polynomial model

nisu značajno razlikovala. Model A sa nižim RMSE se preporučuje sa GPS određenim elipsoidnim visinama kao alternativom konvencionalnom nivelmanu za ortometrijsko određivanje visine u okviru Akure za inženjerske i ekološke aplikacije.

Ključne riječi: DGPS, elipsoidne visine, najmanji kvadrati, ortometrijske visine, polinomski model

1 INTRODUCTION

One of the geodetic framework elements is the height, which might be normal, dynamic, ellipsoidal or orthometric (Sanchez and Sideris, 2017). The height of a point on the earth's surface measured along the plumbline, normal to the Geoid is known as orthometric height. In most engineering, surveying as well as other geospatial applications, orthometric heights relative to the Geoid are required (Erol and Celik, 2004). For over a century, conventional spirit levelling has been a technique of choice in determining orthometric heights due to its simplicity, effective operation, and attainment of remarkable precision (Aleem et al., 2011) i (Isioye et al., 2011). However, the inherent weaknesses of spirit levelling techniques such as operational cost, labour requirement, observational time, prone to systematic errors necessitated further search, which fortunately was provided by the development and application of space technique for Military navigations in point positioning ability (Oluyori et al., 2018).

The advent of satellite-based measurement systems such as the Global Navigation Satellite System (GNSS) has brought tremendous changes in point's position determination. The many benefits offered by Global Positioning System (GPS) have made it a suitable alternative for orthometric height determination over conventional spirit levelling. The GPS gives elevations above a reference ellipsoid, World Geodetic System 1984 (WGS84) (Isioye et al., 2011); thus, it provides the ellipsoidal height. Such geometric height can be transformed into orthometric height if the geoidal height normally derived from a gravimetric geoid of the area is known (Abdullah, 2010) or by adopting global geoid model such as Earth Geopotential Model 2008 (EGM2008), EGM96, EGM84. However, global geoid models are too generalized for local applications which points to the need for local geoid in this case (Odera and Fakuda, 2015). The fundamental relationship that binds the ellipsoidal height from GPS observations and orthometric height from conventional spirit levelling and the geoid undulation is given by Abdullah (2010), Ono (2009), Aleem et al. (2011), Isioye et al. (2011), Eteje et al. (2018) and Oduyebo et al. (2019) as:

$$H = h - N \tag{1}$$

where h represents the ellipsoidal height, H as orthometric height, and N as geoidal undulation. The ellipsoid has a known mathematical surface while the geoid surface is the surface of reference being developed from geoid modelling.

The most precise method for obtaining accurate geoidal height is through the combination of gravimetric observations with a geopotential model. Numerical integration of gravimetric observations using Stoke's integral provides a local gravimetric geoid solution (Abdullah, 2010). However, precise gravimetric geoid for Nigeria has not yet been computed due to the difficulties related to gravity data coverage in most of the country due to the topography. The gravimetric geoid model for Nigeria was investigated by Ezeigbo (1990) and the accuracy obtained was on the meter level, which is inadequate for local applications. Okiwelu et al., (2011) determined geoid undulations for Nigeria using the geopotential model (EGM2008); the study revealed that the Nigerian geoid undulation is affected by a combination of irregular topography and heterogeneous internal mass distribution of the Earth. Epuh et al., (2016) in their study on geoid determination of Gongola basin using isostatic models and seismic reflection data and geophysical interpretation reported a 2.2 m difference using GPS and levelling. Kyamulesire et al., (2020) compared three plane geoid surfaces using geometric approach for orthometric heights modelling in Kampala, Uganda and the result obtained show that model 2 with 4.4 cm is most suitable for orthometric heights interpolation in the study area. A regional geoid model was developed for Jordan by Omar and Abdulla (2007) using gravity data and compared with the GPS/levelling measurements yielding an accuracy of about 40 cm. In another study conducted by Manisa et al. (2016), a second-degree polynomial model was employed to develop a local geoid model from GPS/levelling observations for an area of 100 km x 100 km in Botswana yielding an accuracy of 20 cm. Rabindra et al. (2018) also developed a local geoid model from GPS/levelling observation for Madang town in Papua New Guinea by employing various polynomial models and obtained an accuracy of 20 cm for the third-degree polynomial.

The absence of a national geoid model in Nigeria remains a great drawback to develop local geoid for local application in place of global geoid models. Thus, when no local gravimetric geoid solution is available, a local geoid surface fitting solution may be employed to develop geoid model using geometric approach (which involve the use of ellipsoidal heights obtained from the GPS and orthometric heights obtained through levelling operation to determine the geoid undulation) for local applications. This study, therefore, focuses on the accuracy assessment of three polynomial surfaces for geoid modelling from GPS/levelling observations in Akure with a view to recommend which model to adopt by the GPS user community for various applications.

1.1 Overview of Polynomial Models under Investigation

According to Kirici and Sisman (2017), polynomials can be represented in the form below:

$$N_{x,y} = \sum_{i=0}^{m} \sum_{i=0.j=k-1}^{n} a_{i,j}\, x^i y^j \tag{2}$$

where; $a_{i,j}$ denote polynomial coefficients, m represents the degree of the polynomial and x, y is the plane coordinates.

With respect to expression (2), several polynomial models can be formed subject to the number of known horizontal control points available. For this study, three models were used, and they take the following form:

Model A:

$$N = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 xy + a_6 x^2 y + a_7 xy^2 + a_8 x^2 y^2 \quad (3)$$

Model B:

$$N = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 xy + a_6 x^2 y + a_7 xy^2 + a_8 \Delta h \quad (4)$$

Model C:

$$N = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 xy + a_6 x^2 y + a_7 xy^2 + a_8 x^2 y^2 + a_9 x^3 + a_{10} y^3 . \quad (5)$$

Model A has degree 3 and 4 and is a function of the 2-Dimensional positions i.e., easting (x) and northing (y) of observed points during data acquisition. Model B has degree 3 and is a function of 3-Dimensional positions i.e., easting (x), northing (y), and ellipsoidal height differences (Δh) between average ellipsoidal height and ellipsoidal height of each observed points. The addition of the height term (Δh) in model B is based on the assumption that the geoidal undulation generally follows approximately the topography of the area. Model C has terms of degree 4 and is also a function of the 2-Dimensional positions i.e., easting (x) and northing (y) of observed points during data acquisition.

1.2 Study Area

The study area is Akure, Ondo State in the South-Western part of Nigeria. It lies between latitude 7° 15' N to 7° 30' N and longitude 5° 15' E to 5° 25' E. The topography of the basement complex terrain of the study area is generally undulating with a virtually rugged terrain consisting of hills and valleys. The twenty-one (21) network of control points selected for observations is located in Akure South Local Government Area of Ondo State. Figure 1 (Tata and Ono, 2018) shows the location of the study area and Figure 2 shows the distribution of the control points in the study area.

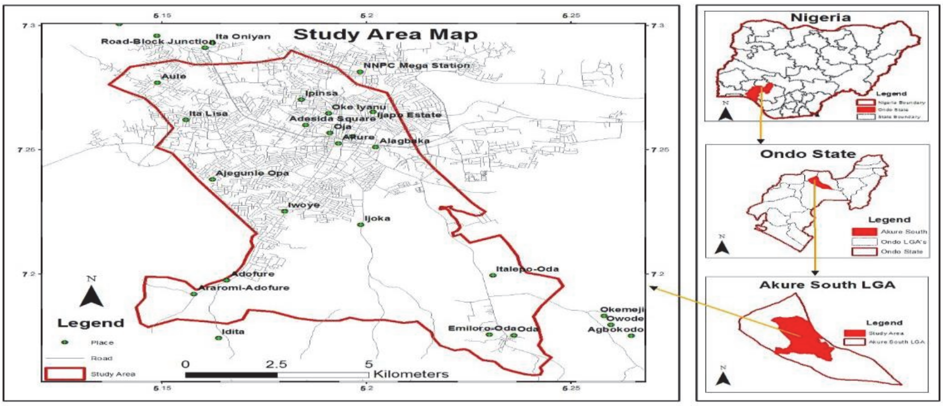

Figure 1. Map of the study area (Source: Tata and Ono, 2018)

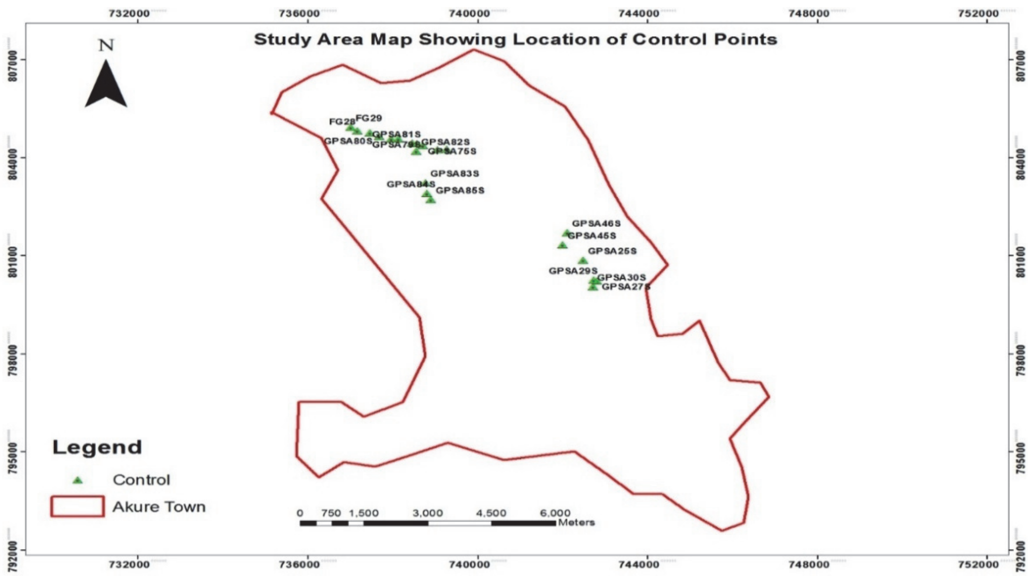

Figure 2. Map of the study area showing the distribution of control points

2 METHODOLOGY

To assess the accuracy of the method of height determination, South Differential Global Positioning System (DGPS) receiver with accessories was used in static mode (2 hours) per stations with five seconds epoch rate for the determination of the 3-dimensional positions (x, y and h) of the selected control points in the study area. The existing/known orthometric heights of the selected control points were obtained from the Department of Surveying and Geo-informatics, FUTA, Akure. They were observed using Leica Jogger 20/24/28/32 modern automatic level (accuracy of ± 2 to 2.55 mm) based on Minna datum which is the Nigeria local vertical datum and computed using the height of collimation method.

2.1 Data Processing

The GPS observed data were post-processed using South DGPS Processor software to obtain the 3-Dimensional coordinates (easting, northing, and ellipsoidal height) of the points based on UTM projection system. The geoid heights of the control points used for the local geometric geoid modelling, were computed by finding the differences between the ellipsoidal heights from GPS observation and the known orthometric heights of the points. Table 1 shows the results of the post-processed 3-D coordinates and computed geoid undulation.

Table 1.
Post-processed 3-D co-ordinates (eastings, northings, and ellipsoidal heights) and geoid undulations

Control Point	Easting X (m)	Northing Y (m)	Ellipsoidal Height h (m)	Horizontal accuracy (m)	Vertical accuracy (m)	Orthometric Height H (m)	Geoid Undulation $N = h - H$ (m)
GPSA72S	739267.633	804257.824	359.913	0.001	0.002	346.470	13.443
GPSA73S	739062.214	804248.124	358.605	0.001	0.002	345.146	13.459
GPSA75S	738717.518	804372.834	352.048	0.002	0.003	338.388	13.660
GPSA76S	738461.436	804447.291	350.313	0.003	0.004	336.666	13.647
GPSA77S	738139.272	804573.472	348.284	0.003	0.005	334.651	13.633
GPSA78S	737957.029	804568.798	351.006	0.002	0.003	337.365	13.641
GPSA79S	737685.594	804643.672	356.321	0.002	0.003	342.538	13.783
GPSA80S	737459.700	804758.843	359.479	0.002	0.004	345.831	13.648
FG28	737176.503	804822.180	359.507	0.003	0.005	345.817	13.690
FG29	737005.872	804931.255	353.718	0.003	0.005	340.115	13.603
GPSA81S	738575.104	804424.237	351.926	0.003	0.004	338.215	13.711
GPSA82S	738560.193	804192.634	347.812	0.003	0.005	334.102	13.710
GPSA83S	738770.801	803231.350	363.926	0.002	0.004	349.765	14.161
GPSA84S	738804.396	802904.596	360.031	0.002	0.004	345.840	14.191
GPSA85S	738896.796	802709.338	353.403	0.003	0.005	339.197	14.206
GPSA45S	741997.479	801334.010	347.433	0.005	0.006	332.621	14.812
GPSA46S	742108.308	801695.041	346.735	0.003	0.004	331.915	14.820
GPSA25S	742487.484	800866.014	346.675	0.005	0.008	332.413	14.262
GPSA27S	742808.473	800214.552	355.349	0.006	0.007	341.160	14.189
GPSA29S	742728.553	800246.724	356.038	0.006	0.009	342.644	13.394
GPSA30S	742711.597	800052.122	358.823	0.007	0.008	345.019	13.803

2.2 Solving the Models using Least Squares Method

The method of least squares is a standard approach in an over-determined system to the approximate solution. In this study, the method of least-squares has been employed to determine the parameters in Model A, Model B, and Model C.

The least-squares method is based on the minimization of the sum of the squares of residuals to be a minimum. The solution of the formulation of the least-squares is given by Ono (2002) and Oluyori et al. (2018) as:

$$X = (A^T PA)^{-1}(A^T PL) \quad (6)$$

$$X = (A^T A)^{-1}(A^T L) \quad (7)$$

Equation (7) is for unit weight due to equal reliability of observations.

Using the mathematical model of observation method, we have the expression below:

$$V = AX - L \quad (8)$$

where V is the vector of residuals and A is the design matrix, X is the vector of unknown parameters and L is the vector of observations (i.e., $N = h - H$).

The computed geoid heights and the positions of the points were applied in equations (3) and (5) to obtain the model parameters of Models A and C. While the parameters of model B, were computed with the geoid heights, positions and the ellipsoidal heights of the points using equation (4). The constants for each of the models were determined with the least-squares method using MATLAB software. The values of the constants for Model A, B, and C, i.e., equation (3), (4) and (5) is shown in Table 2.

Table 2.
Values of Models A, B and C Estimated Parameters

Parameter	Model A Parameter value (m)	Model B Parameter value (m)	Model C Parameter value (m)
a_0	-13163694.598671418	-8099337.23786035	15514537841.85525
a_1	23.5609239181734954	28.627740536572667	-41699.2898744777
a_2	19.92892557504632237	3.0795819221036249	-39092.9375029699
a_3	-0.00000851838866977	-0.000024402999582	0.027929359470683
a_4	-0.00000454837117535	0.000008584387036	0.024547482254017
a_5	-0.00002454294958114	-0.000025560598218	0.105241598654605
a_6	-0.00000000000143775	0.0000000000301824	-0.000000071064560
a_7	-0.0000000000558305	-0.000000000012183	-0.00000065867041
a_8	0.0000000000000000147	0.00444004031072520	0.0000000000000444
a_9			0.0000000020546218
a_{10}			-0.0000000000672036
	σ = 0.147 m	**σ = 0.161 m**	**σ = 0.149 m**

2.3 Determination of Geoid Surface using Polynomial models

Microsoft Excel 2010 facilities were used to compute the polynomial models to determine the geoid undulation (N) and orthometric height (H) for the twenty-one (21) selected contr ol points used in the study. The computed parameters and the coordinates of the points were used in the

Microsoft Excel programs developed using equations (3) and (5) (model A and C respectively) while the computed parameters, coordinates and the ellipsoidal heights of the points were used in equation (4) (model B). The statistics for the absolute differences between the geoidal undulation obtained from the three models and GPS-derived values at the GPS/levelling points is given in Table 3. The models' orthometric heights, were obtained by finding the differences between the ellipsoidal and the models' geoid height. The results of the existing orthometric heights and computed orthometric heights for model A, model B, and model C are shown in Table 4.

Table 3.
The Statistics of Differences between Geoidal Undulation from the three polynomial models (A, B and C) and GPS for the Selected Control Points

	N^{GPS} (1)	$N^{Model\ A}$ (2)	$N^{Model\ B}$ (3)	$N^{Model\ C}$ (4)	Residuals 1 - 2	1 - 3	1 - 4
Maximum value (m)	14.820	14.815	14.714	14.843	0.318	0.356	0.351
Minimum value (m)	13.394	13.492	13.492	13.403	-0.433	-0.487	-0.459
Standard deviation (m)	0.409	0.381	0.376	0.381	0.147	0.161	0.149
No of stations = 21							

Table 4.
Existing orthometric heights and computed orthometric heights for Models A, B and C

Control Points	Existing Orthometrc Height, *H* (m)	Model Orthometric Height: Model A, *Ha* (m)	Model B, *Hb* (m)	Model C, *Hc* (m)	Difference: (*H* – *Ha*) (m)	(*H* – *Hb*) (m)	(*H* – *Hc*) (m)
GPSA72S	346.470	346.475	346.421	346.510	-0.005	0.049	-0.041
GPSA73S	345.146	345.091	345.018	345.036	0.056	0.128	0.111
GPSA75S	338.388	338.411	338.447	338.445	-0.023	-0.059	-0.057
GPSA76S	336.666	336.561	336.693	336.665	0.105	-0.027	0.001
GPSA77S	334.651	334.720	334.667	334.604	-0.069	-0.016	0.047
GPSA78S	337.365	337.229	337.346	337.322	0.136	0.019	0.043
GPSA79S	342.538	342.734	342.665	342.668	-0.196	-0.127	-0.130
GPSA80S	345.831	345.856	345.846	345.794	-0.025	-0.015	0.037
FG28	345.817	345.871	345.832	345.923	-0.054	-0.015	-0.106
FG29	340.115	339.997	340.017	340.010	0.118	0.098	0.105
GPSA81S	338.215	338.298	338.328	338.306	-0.083	-0.113	-0.091
GPSA82S	334.102	334.064	333.992	334.009	0.038	0.110	0.093
GPSA83S	349.765	349.740	349.759	349.750	0.025	0.006	0.015
GPSA84S	345.840	345.771	345.849	345.883	0.069	-0.009	-0.043
GPSA85S	339.197	339.266	339.191	339.167	-0.069	0.006	0.030
GPSA45S	332.621	332.747	332.719	332.747	-0.126	-0.098	-0.126
GPSA46S	331.915	331.920	332.025	331.892	-0.006	-0.110	0.022
GPSA25S	332.413	332.328	332.220	332.353	0.085	0.193	0.060
GPSA27S	341.160	341.478	341.516	341.511	-0.318	-0.356	-0.352
GPSA29S	342.644	342.211	342.157	342.185	0.433	0.487	0.459
GPSA30S	345.019	345.112	345.172	345.100	-0.092	-0.152	-0.080
			***RMSE* (m) =**		**0.143347**	**0.157412**	**0.145137**
			R^2		**0.870795**	**0.844254**	**0.867544**

2.4 Determination of Goodness of Fit

The goodness of fit of the models was determined using the coefficient of determination (R^2) which is expressed as:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} V_i^2}{\sum_{i=1}^{n}\left(q_i - \overline{q}_i\right)^2} \tag{9}$$

Where $\sum_{i=1}^{n} V_i^2$ is the sum of the squared residuals adjusted for each of the stations in the fit and $\sum_{i=1}^{n}\left(q_i - \overline{q}_i\right)^2$ is the sum of the squared differences between the original height misclosures $q_i = (h - H)$ and their mean value $\overline{q}_i$. This statistical measure R^2 takes values between 0 and 1, with values closer to 1 implying a better fit. Since the coefficient of determination (R^2) for each of the models is closer to 1 as shown in Table 4, it indicates how well the models predict the geoid undulation and the orthometric height.

2.5 Root Mean Square Error

The accuracy and reliability of the model were also accessed using the Root Mean Square Error (*RMSE*). *RMSE* is a statistical tool used in measuring the difference between values predicted by a model and the values observed from the environment or data set being modelled. Then, if X_{obs} is the existing orthometric height of the control points and X_{model} is the estimated orthometric height of the control points using the polynomial models, estimates of the root mean square residual is given as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(X_{obs,i} - X_{model,i})^2}{n}} \tag{10}$$

Where X_{obs} is the existing values, X_{model} is the modelled values, *n* is the number of points.

The smaller the value of the root mean square error estimate the better the accuracy attainable with the orthometric heights obtained from the models. The *RMSE* for Model A is 14.3 cm, model B is 15.7 cm and model C is 14.5 cm, which indicates that the three models are of comparable accuracies and can be used interchangeably for the determination of orthometric heights in the study area by GPS users.

The standard error value computed and compared within the permissible limits given by American Society of Photogrammetry and Remote Sensing (ASPRS 1993) specifications as shown in Table 5 for topographic elevation accuracy requirement.

Table 5.
ASPRS Topographic Elevation Accuracy Requirement for Well-Defined Points (Source: American Society of Photogrammetry and Remote Sensing (ASPRS 1993))

Contour Interval (m)	Class I (m) High Accuracy/Standard Error Accuracy	Class II (m) Standard Error	Class III (m) Standard Error
0.5	0.08	0.16	0.25
1.0	0.17	0.33	0.5
2.0	0.33	0.67	1.0
4.0	0.67	1.33	2.0
5.0	0.83	1.67	2.5

From Table 5, it is seen that, it is possible to use the three models to produce contours of 1m interval for engineering and environmental applications. In view of ever-increasing demand of large-scale maps with contour interval of 1 m, the space-based technology of GNSS or GPS will be a major contributor in providing precise locations including heights.

2.6 Analysis of Variance (ANOVA) Test

In this study, ANOVA one-way was used to test whether there are any significant differences in the performance of the models based on the mean orthometric heights of the three models. The ANOVA test is stated below:

- H_0 = mean H of model A is equal to mean H of model B is equal to mean H of model C
- H_1 = mean H of model A is not equal to mean H of model B is not equal to mean H of model C

The decision rule is given as if $F_{cal} > F_{tab}$ at 0.05 significant level, reject H_0 and accept H_1

Since $F_{cal} < F_{tab}$ i.e., 0 is less than 3.150 we accept H_0 which means there are no significant differences between the orthometric heights of the three models.

3 ANALYSIS OF RESULTS

The parameters for each of the three polynomial models were determined with the least square method and the result is presented. The numerical results for the three models as shown in Table 2 revealed that, model A gave the best fitting for geoid modelling in the study area with minimum standard deviation of 0.147 m compared to model B and model C that gave 0.161 m and 0.149 m respectively. The coefficient of determination was used to determine the goodness of fits of each of the models for which values closer to 1 was obtained which indicates that the three models are of adequate predictive capacity for the geoid undulation and orthometric height. It can be seen in Table 3 that the minimum and the maximum orthometric heights of the three polynomial geoid models, Model A, Model B and Model C are respectively 331.920 m and 349.740 m, 332.025 m and 349.759 m, and 331.892 m and 349.750 m which implies that orthometric heights can be respectively obtained using the three models (Models A, B and C) in the study area. Root Mean Square Error (*RMSE*) was also used to assess the accuracy of the three models in which model A has the lowest *RMSE* value of 14.3 cm while model B has RMSE value of 15.7 cm and model C has RMSE value of 14.5 cm, which shows that model A is better and most suitable for application in the study area than the two other models. It is worth to mention that this level of accuracy is reasonably fair considering the achievable accuracy of 5 – 10 cm in case of EGM2008 (Nikolaos *et al.,* 2012).

The RMSE values of model A and model C based on 2-D position indicate that the two models are superior for orthometric height determination using DGPS relative technique than model B that is based on 3-D positions. Also, the RMSE value of model B indicates that the inclusion of height term in the model does not contribute any significant improvement to the results compared to those given by model A and model B. In a previous study conducted by Oluyori *et al.,* (2018), a standard error of 11 cm and 13 cm were obtained for the 2-D and 3-D position polynomial model adopted for orthometric height modelling in FCT Abuja, Nigeria. Rabindra *et al.,* (2018) also employed various polynomial models to develop a local geoid model from GPS/levelling observation for Madang town in Papua New Guinea and obtained an accuracy of 20 cm for the third-degree polynomial. However, to achieve still higher accuracy, number of control points may be increased with increased in area so that it is with uniform distribution spatially and topographically.

The one-way ANOVA test conducted for comparison of the three models showed acceptance of the null hypothesis H_0 which indicates that there are no significant differences between the means of the models. This denotes that, polynomial models which are a function of 2-D (x, y) and 3-D (x, y, h) positions are suitable for the development of geometric geoid within the study area.

4 CONCLUSIONS

The paper has investigated the accuracy of three polynomial models for orthometric height determination in Akure. The ellipsoidal height (h) obtained from field observation was used in conjunction with the existing orthometric height of twenty-one (21) control points to compute the geoid undulation of each point. The result obtained from the study shows that model A gives better result with an *RMSE* value of 14.3 cm than model B and model C. The inclusion of height term in model B did not add any significant improvement to the accuracy of orthometric height determination compared to model A and model B. Considering the result obtained from the analysis, the use of the polynomial model (such as model A) is adequate in providing estimated orthometric height. Thus, for engineering, geophysical, and other geospatial applications, model A developed with DGPS ellipsoidal height will serve as a potential replacement for conventional spirit levelling for orthometric height determination in the study area.

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